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Robust Charge Mix Optimization for Cast Iron Foundry

Deepak Chowdhary, Anirudh Chowdhary, Rahul V, Abhishankar Kumar, Nilanjan Banerjee
MPM Infosoft Pvt. Ltd., Chennai, Tamil Nadu, India

Nabil Magbool Jan
Indian Institute of Technology Tirupati, Yerpedu, Andhra Pradesh, India

ABSTRACT

Optimizing the charge mix in a cast iron foundry is challenging due to the uncertainty in the chemical composition, which impacts tensile strength and hardness, posing risks to meeting composition constraints and casting quality. This paper discusses a novel approach to characterize the uncertainty in scrap composition while assuming the probabilistic distribution of the uncertainty and formulate it as a robust optimization problem. The optimization problem aims to minimize the cost of scrap materials by determining the optimal scrap weight, subject to constraints that ensure the chemical composition stays within specified limits. Constraints are included to ensure that the desired tensile strength and hardness are achieved consistently. The proposed robust formulation is validated using a Monte-Carlo simulation setup with 10^6 different realizations of scrap chemical composition. The simulation results show that the nominal formulation meets composition constraints with only a 1% probability, while the robust formulation achieves a 99% probability of satisfying these constraints.

Keywords: charge mix, cast iron, scrap composition, robust optimization, linear programing, simulation

INTRODUCTION

Foundries play an important role in the world of manufacturing, producing various products for different industries including automotive, aerospace, construction, and equipment. Cast iron foundries face unique challenges because of the involvement of a variety of processes and alloy elements used. Cast iron consists of various quantities of alloying elements (i.e., silicon, carbon manganese etc.) based on the casting alloy specifications. However, achieving a high-quality cast product with a low production cost requires proper selection and formulation of the charge mix. A charge mix is the combination of raw materials added to the melting furnace to produce the final casting. Normally, the composition of a charge mix in a foundry includes pig iron, high-carbon scrap, low-carbon scrap, ferroalloys, and other alloying elements. They are mixed in different proportions according to the material property requirement of the final casting. The quantity of each alloying element affects the mechanical, thermal, and chemical properties of the cast sample. It should be noted that the price of each charge mix ingredient varies considerably therefore the foundry carefully chooses the quantity of each ingredient that will help reduce the production cost of the cast sample without affecting the final desired properties. Because of the volatility of the raw material costs, availability, and quality, optimizing the charge mix is a complex and challenging task.

Conventional methods generally used by the foundries for optimizing the charge mix include empirical rules, heuristic approaches, trial-and-error experimentation, or experiential knowledge. Although in some instances, these methods may provide reasonable results, they may not be efficient in consistently providing the best possible outcomes. Furthermore, they may not be efficient enough to account for the uncertainties associated with raw material costs and availability which may result in financial losses. The target or desired properties associated with the casting provides additional constraints that need to be addressed by the metalcaster when choosing raw materials for the charge. Hence, in this regard the development of a robust linear programming (RLP) approach can provide significant advancement. This approach can be considered as an advanced version of the conventional linear programming method that accounts for the uncertainty in the input data.[1] It guarantees that the solution remains feasible and near-optimal, even when the input data varies within the specified bounds by including robustness in the optimization model formulation. This formulation technique can be highly beneficial in cast iron foundries where uncertainties in the raw material costs and availability can significantly impact the cost-effectiveness of the charge mix which may eventually impact the casting quality. By using RLP, foundries can develop strategies for the charge mix preparation that can be resilient to these fluctuations, thereby minimizing costs while maintaining product quality. The rapid development of computational techniques over the last few decades has prompted researchers to investigate and find solutions to many industrial problems using data analytics and optimization. Prevalent use of linear programming and its various variants can be seen, specifically, in the field planning and scheduling of various foundry processes to achieve different goals like manufacturing cost reduction, increasing process efficiency and improving energy efficiency.

Ramin et al.[2] developed a novel methodology to utilize the demand flexibility for energy-intensive industries and get benefited from the demand-side management programs and reserve markets. Two separate demand response optimization problems for multistage and multi-line energy-intensive processes were considered because of the unique features and economic priorities associated with this market. The mixed integer linear programming method was used to develop the mathematical framework considering the power flexibility and storage buffering capacity. It was emphasized that the proposed framework will minimize consumption costs and enable participation in energy and reserve markets. By extending the model to account for imbalance costs and maximizing reserve capacity benefits, the framework offers bidding optimization for each production line.

Sakalli and Birgören[3] developed a mathematical framework to optimize jointly cost and quality by developing a chance-constrained mathematical model for maximizing the minimum process capability level for a fixed cost. By analyzing the data from a brass manufacturer, it was observed that the frontier was highly nonlinear which allows the decision-makers to choose a competitive process capability and cost valuation in combination. It was emphasized that the developed method can be made applicable to different blending situations where statistical variability exists.

The work of Suthar et al.[4] focused on improving quality in the metalcasting industry by analyzing surface roughness, mechanical properties, dimensional accuracy, and defects in cast parts. The machine learning model was developed by considering the importance of each parameter. The inputs from the foundry experts were also considered in the model development process. The study outlines the potentially important process variables and also uses data from the experts to understand their effect on the quality aspects of cast products. Baitiang et al.[5] implemented a methodological framework for data-driven process control to improve the quality of cast samples for a cast iron foundry. Industrial production data that was used for this purpose was from the foundry that manufactures automotive and oven parts. Various statistical tools were used to identify critical parameters. A regression model technique was used to predict and control molding sand moisture and liquidus temperature of the melt. The developed model was useful in production control and process input correction to achieve target cast quality. Chowdhary et al.[6] used a linear programming approach to minimize the cost of charge mix melt preparation for the alloy FG-220. But in their work, the surety of achieving the target chemical composition under various fluctuations in input variables has not been considered, which lacks its applicability in real shop floor scenarios.
In the present work, the mathematical optimization formulation for charge mixes when preparing cast iron samples is presented. The works related to the development of optimal "charge-mix prescription" by considering variety of scenarios, like achieving the required target chemistry and considering the availability of raw materials etc. have not been reported and discussed in detail. Therefore, we propose a methodology that can give insight into how these various fluctuations in the input parameters can be accommodated so that the cost-effective charge mix can be achieved without degrading the material properties and target chemistry of the cast samples.

MATHEMATICAL FORMULATION

In this section, we present two mathematical optimization formulations for charge mix optimization in cast iron foundries.

NOMINAL OPTIMIZATION FORMULATION

In this section, a nominal optimization model for charge mixing is proposed. The primary purpose of charge mixing is to blend different scrap materials in some proportions such that the desired chemical composition and mechanical properties (such as tensile strength and hardness) are attained. As the cost of various scarp materials are different, it is imperative to minimize the total cost of scrap materials while enforcing the desired properties, and available inventories as constraints. Firstly, we propose the details of constraints, followed by the overall optimization formulation.[6]

Target Chemistry Constraint:

Ensures that the final chemistry of the cast sample is within the specified limit according to the need of the alloy. Let us denote $x \in \mathbb{R}^{n\times 1}$ as mass of n scrap, M_t represents the total furnace capacity, $j \in \mathbb{R}^{n\times r}$ as the composition matrix of n scrap and r species, and $R \in \mathbb{R}^{n\times r}$ as the recovery matrix, respectively. Now, defining $J_{n\times r}$ $(=j_{n\times r} \circ R_{n\times r})$ as the Hadamard product of scrap and alloys composition with recovery matrix, we can mathematically represent the target chemistry constraint as follows in Equation 1.

$$l_b \leq \frac{1}{M_t}(J^T x) \leq u_b \quad \textbf{Eqn. 1}$$

Furnace Capacity Constraint:
Defines the melt amount that can be made on the furnace, and it is expressed in Equation 2.

$$x^T 1 = M_t \quad \textbf{Eqn. 2}$$

Inventory Constraint:
Denoting I_i is the amount of scrap i available in the inventory $i \in \{1, \ldots, r\}$, we can express the amount of scrap available in the foundry as in Equation 3.

$$x_i \leq I_i \quad \textbf{Eqn. 3}$$

Tensile Strength Constraint:
The purpose of this constraint is to ensure that the tensile strength of the cast sample stays within the prescribed value (t_r) of the final cast product, and it is given[7] in Equation 4,

$$f_T(J^T x) \geq t_r \quad \textbf{Eqn. 4}$$

where, $f_T(J^T x) = 1049.41 - 209.15 \times \frac{1}{M_t}\left((J^T x_c) + 0.25(J^T x_{Si}) + 0.5(J^T x_P)\right)$

Hardness Constraint:
This constraint ensures that the hardness of the final cast sample stays within the prescribed range $[t_{h,min}, t_{h,max}]$ of the final cast product, and it is given in Equation 5.[7]

$$t_{h,max} \geq f_H(J^T x) \geq t_{h,min} \quad \textbf{Eqn. 5}$$

where, $f_H(J^T x) = 559.98 - 91.68 \times \frac{1}{M_t}\left((J^T x_c) + 0.25(J^T x_{Si}) + 0.5(J^T x_P)\right)$

Scrap Constraint:
It represents the amount of scrap allowed in each charge for producing cast samples (Equation 6).

$$x_{i,min} \leq x_i \leq x_{i,max}; \quad \textbf{Eqn. 6}$$

where $x_{i,min}$ and $x_{i,max}$ are the min and max amount of scrap (i) should be added to the charge. $i \in \{1, \ldots, r\}$

Low Carbon Constraint:
It is required to ensure that the amount of low carbon scrap that should be used for a given grade of the material (Equation 7).

$$lcs_{min} \leq \sum_{\forall i \, in \, low \, carbon} x_i \leq lcs_{max} \quad \textbf{Eqn. 7}$$

where, lcs_{min} and lcs_{max} are the lower bound and upper bound of the total low carbon scrap.

Let us denote $c \in \mathbb{R}^{n\times 1}$ is a vector of cost per kg of scraps. Now, the overall optimization formulation for charge-mixing can be cast as follows (Equation 8):

$$\min_x c^T x \;\; s.t \begin{cases} l_b \leq \frac{1}{M_t}(J^T x) \leq u_b \\ x^T 1 = M_t \\ x_i \leq I_i \\ f_T(J^T x) \geq t_r \\ t_{h,max} \geq f_H(J^T x) \geq t_{h,min} \\ x_{i,min} \leq x_i \leq x_{i,max} \\ lcs_{min} \leq \sum_{\forall i \, in \, low \, carbon} x_i \leq lcs_{max} \end{cases}$$

Eqn. 8

STOCHASTIC OPTIMIZATION FORMULATION
The objective of this work is to develop a mathematical framework to characterize the uncertainty associated with the scrap composition while assuming a probabilistic distribution and developing it as a stochastic optimization problem. The optimization framework seeks to reduce the cost of scrap materials by estimating the ideal scrap weight while adhering to constraints that ensure the chemical composition remains within the specified limits. Additionally, constraints are used to guarantee that the specified tensile strength and hardness are constantly obtained within the desired limit.

Target Chemistry Constraint:
Ensures that the final chemistry of the cast sample remains within the limits of the specifications considering the uncertainty in chemical composition of scrap materials, and it can be expressed as (Equation 9),

$$l_{b,i} + \beta \frac{||P_i x||_2}{M_t} \leq \frac{(J^T x)_i}{M_t} \leq u_{b,i} - \beta \frac{||P_i x||_2}{M_t} \quad \forall i \in \{1, \ldots, r\}$$

Eqn. 9

where $l_{b,i}$ and $u_{b,i}$ denote the lower limit and upper limit of required chemistry for r chemical components, respectively.

Tensile Strength Constraint:
Defining $\tilde{P} \in \mathbb{R}^{n\times n}$, tensile strength constraint to account for uncertainty in chemical composition is expressed as (Equation 10),

$$f_T(J, x, \tilde{P}, \alpha_t) \geq t_r \quad \textbf{Eqn. 10}$$

where $\tilde{P} = \sum_{i=1}^{r} \alpha_i P_i$ and $P_i = \mathrm{diag}(\Delta J_i) \forall i \in \{1, \ldots, r\}$ is a diagonal matrix n $\times\, n$ have standard deviation of element i in every scrap, and $\alpha_{t\, 1\times r}$ represents the coefficient vector of tensile strength function.

Hardness Constraint:
Similarly, the purpose of this constraint is to ensure that the Hardness of the cast sample attains the required needed by the foundry (Equation 11),

$$t_{h,\, min} \leq f_H(J, x, \tilde{P}, \alpha_h) \leq t_{h,\, max} \quad \textbf{Eqn. 11}$$

Now, the overall optimization problem that minimizes the cost of charge-mixing in a cast iron foundry considering uncertainty in chemical composition in the scarp materials is presented as follows (Equation 12),

$$\min_{x} c^T x \; s.t \begin{cases} l_{b,i} + \beta \frac{||P_i x||_2}{M_t} \le \frac{(J^T x)_i}{M_t} \le u_{b,i} - \beta \frac{||P_i x||_2}{M_t} \\ \forall i \in \{1, \ldots, r\} \\ x^T 1 = M_t \\ x_i \le I_i \\ f_t(J, x, \tilde{P}, \alpha_h) \ge t_r \\ t_{h,max} \ge f_H(J, x, \tilde{P}, \alpha_h) \ge t_{h,min} \\ x_{i,min} \le x_i \le x_{i,max} \\ lcs_{min} \le \sum_{\forall i \; in \; low \; carbon} x_i \le lcs_{max} \end{cases}$$

Eqn. 12

SOLUTION METHODOLOGY

The robust optimization problem was formulated and solved using the CVXPY library, a powerful tool for convex optimization. The problem incorporated uncertainties in scrap composition, modelled through second-order cone constraints, to account for potential deviations from nominal values. The decision variable was the scrap mix, and the objective was to minimize the cost of the charge subject to compositional constraints and furnace capacity limits. A key component of the formulation was the introduction of the strictness factor (β), which adjusted the conservatism of the solution by controlling the degree of robustness against uncertainties. As β increased, the constraints became tighter, reducing the risk of violation while increasing the cost. The optimization problem was then solved using CVXPY's efficient solvers, which handles large-scale convex programs. The framework allowed for seamless integration of uncertainty into the problem, ensuring a balance between cost minimization and constraint satisfaction. The flexibility of CVXPY enabled rapid prototyping and adjustments to the model, making it a highly effective approach for solving robust optimization problems in industrial applications.

CASE STUDY

In the present work, a case study was considered for the production of cast iron FG-241. The minimum requirement of the tensile strength of the sample is 241MPa. The desired chemical composition of the sample is as follows:
$q = [C \;\; Si \;\; Mn \;\; Cr \;\; P \;\; S \;\; Sn]$,
$l_b = [3.25 \;\; 1.9 \;\; 0.65 \;\; 0.25 \;\; 0.04 \;\; 0.06 \;\; 0.065]$,
$u_b = [3.35 \;\; 2.1 \;\; 0.75 \;\; 0.35 \;\; 0.08 \;\; 0.08 \;\; 0.085]$.

The cost of each raw material in ₹/kg is given by
$c = [64 \;\; 53 \;\; 53 \;\; 53 \;\; 58 \;\; 53 \;\; 75 \;\; 153 \;\; 110 \;\; 50 \;\; 130 \;\; 2500]$
The chemical composition of the raw material that is used in preparing the charge-mix is mentioned in Table 1.

Table 1. Chemical Composition of Raw Materials

Material	C	Si	Mn (%)	P	S	Cu (%)	Cr	Sn (%)
PIG IRON	4.18	2	0.6	0.06	0.007	0.006	0	0.003
M.S.Scrap	0.16	0.28	0.8	0.05	0.01	0.005	0	0.001
in coated scrap	0.1	0.01	0.35	0.03	0.01	0.01	0	0.04
Silicon Punching scrap	0	1.2	0.17	0.02	1E-04	0.01	0	0.003
C.I Boring	3.1	1.8	0.65	0.05	0.07	0.02	0.2	0.06
Runner Raiser	3.2	1.8	0.65	0.05	0.07	0.02	0.2	0.06
CPC	85	0	0	0	0	0	0	0
FeSi	0	70	0	0	0	0	0	0
FeMn	0	0	60	0	0	0	0	0
FeS	0	0	0	0	35	0	0	0
FeCr	0	0	0	0	0	0	60	0

RESULTS AND DISCUSSION

In this study, a robust optimization framework was developed to address the challenge of balancing cost minimization and constraint violations in the charge mix for metal casting processes. By incorporating uncertainty in scrap composition, the model provides solutions that are both cost-effective and resilient against violations of target chemistry constraints. The strictness factor, β, was introduced to regulate the level of conservatism in the optimization, with the expectation that as β increases, the system becomes more robust to uncertainties but at the expense of increased cost. The target chemical composition and the achieved chemical composition for various cases of FG-241 are displayed in Table 2. Similarly, the achieved raw material composition (scrap) for various cases and their respective costs are mentioned in Table 3.

INFLUENCE OF β ON COST AND CONSTRAINT VIOLATIONS

Four scenarios were evaluated with β values of 0, 1, 1.96, and 2.86, demonstrating the trade-off between charge cost and constraint violation rates:

- **β = 0**: The lowest value of β represents a purely cost-driven solution, where the cost per kilogram of charge was minimized to 55.636 ₹/kg. However, this came with a constraint violation rate of 99.2107%, indicating that although the cost is minimized, the likelihood of violating compositional constraints is extremely high. This solution is suitable only when cost is the overriding priority, and the risks associated with constraint violations are acceptable.

- **β = 1**: Here, the model showed a slight increase in cost to 55.686 ₹/kg, with a corresponding reduction in constraint violations to 70.213%. While the system became more resilient to violations, the risk was still relatively high, suggesting that this level of conservatism does not sufficiently mitigate the impact of uncertainties in scrap composition.

- **β = 1.96**: Here, a balance between cost and robustness was achieved. The cost increased to 55.735 ₹/kg, but the constraint violation rate dropped significantly to 16.619%. This reduction in violations indicates that the model becomes much more robust against uncertainties, ensuring that material composition constraints are adhered to while keeping the cost increases within reasonable limits.

- **β = 2.86**: In this case, the constraint violation of 1.262 % was achieved but an increase in the cost was observed up to 58.257 ₹/kg. The further decrease in violation suggests that the model has grown considerably more resistant to uncertainty, ensuring that material composition limitations are met.

The results clearly demonstrate the inverse relationship between charge cost and constraint violations. As the strictness factor (β) increases, the system becomes more robust, with a sharp decrease in the likelihood of violating constraints. However, this comes at the cost of a higher charge cost. For instance, moving from β = 0 to β = 2.86 results in an increase in cost by 2.621 ₹/kg, but it achieves a 97.95% reduction in constraint violations.

In real-world applications, the decision to prioritize cost or robustness will depend on the specific operational and quality requirements of the production process. In high-precision environments where composition violations could result in significant quality issues, higher β values would be preferable despite the increased costs. On the other hand, in cost-sensitive environments where minor compositional deviations are acceptable, lower β values may be more appropriate.

Table 2. Comparison of Resulting Chemical Composition of Target Material FG-241 Obtained Using Nominal and Robust Formulation

β	**0**	**1**	**1.96**	**2.86**
C	3.25	3.272	3.293	3.3
Si	1.9	1.911	1.922	1.929
Mn	0.65	0.654	0.658	0.681
P	0.04	0.04	0.041	0.051
S	0.06	0.06	0.061	0.061
Cr	0.25	0.252	0.253	0.255
Sn	0.065	0.065	0.066	0.066
Constr Violation (%)	99.2107	70.2131	16.6191	1.262

Table 3. Comparison of Scrap Weight in % to Furnace Capacity Obtained Using Nominal and Robust Formulation

β	**0**	**1**	**1.96**	**2.86**
Pig iron	5	5	5	19.2
M.S scrap	4.407	5.555	6.656	40
Sn scrap	0	0	0	0
Si Scrap	35.593	34.445	33.344	0
C.I Boring	0	0	0	10
Runner raiser	52.177	52.118	52.062	27.876
CPC	1.604	1.629	1.654	1.449
FeSi	0.602	0.635	0.666	1.073
FeMn	0.308	0.304	0.299	0
FeS	0.065	0.066	0.067	0.083
FeCr	0.212	0.215	0.218	0.276
Sn	0.032	0.033	0.033	0.043
Tensile	266.131	260.96	255.942	253.039
Hardness	216.653	214.386	212.187	210.914
Cost (₹/kg)	55.636	55.686	55.735	58.257

CONCLUSIONS

The robust optimization framework successfully mitigates the trade-off between cost and constraint violations by allowing for the tuning of the strictness factor (β). The results suggest that increasing β improves robustness to compositional uncertainties at the expense of higher costs. The model provides flexibility in selecting the optimal balance between cost and violation rates, depending on the operational priorities. The ability to adjust β according to production requirements offers a valuable tool for foundries and metal casting operations to achieve both cost efficiency and adherence to strict material compositions, ensuring product quality and operational sustainability.

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